Journal of Engineering Physics and Thermophysics, Vol. 73, No. 4, 2000

FLOW OF A HEAVY VISCOPLASTIC FLUID IN A GAP BETWEEN ROTATING ROLLERS

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The problem of the flow of a Shvedov–Bingham fluid in a vertical gap of oppositely rotating rollers with allowance for the intrinsic weight of the fluid is formulated and solved.

The problem under consideration is associated with the process of drying of a highly filled suspension on a two-roller dryer. With a similar mechanism of flow the process of depositing the material being dried on rollers differs significantly from that of calendering of polymer materials, which is very close in scheme. First of all the viscosity of the medium is 2–4 orders of magnitude lower than that of calendered polymers, for example, of rubber mixures. Therefore, whereas power and mechanical calculations are determinable for calenders, for the process of depositing paste-type compositions the efforts appearing during the process are comparatively small and the basic technological parameter affecting the drying rate is the thickness of the material [1]. Furthermore, the forces of viscous friction are comparable to those of the intrinsic weight of the fluid. A detailed review of works devoted to the flow of non-Newtonian fluids in a roller gap is given in [2].

The objective of the work is to analyze the influence of the intrinsic weight and the rheological properties of the fluid on the character of its flow in the gap.

The scheme of flow and the system of coordinates are presented in Fig. 1. We assume that the rollers have sufficient length, thereby neglecting the flow of the material along the rollers (the problem is plane). The rotational speed of the rollers is insignificant and inertial forces are not taken into account. The physical properties of the fluid are independent of temperature and pressure. The magnitude of the minimum interroller gap is small compared to the radius of curvature of the rollers. The medium is described by the Shvedov–Bingham rheological model. The flow direction is from top to bottom.

The origin of a Cartesian coordinate system is placed in the middle of the cross section of the minimum gap. The y axis is directed horizontally, while the x axis is directed vertically downward. The fluid level $x = x_0$ is constant. The peripheral velocity of the rollers is V and their radius is R. The minimum gap between the rollers is $2H_0$ and current gap is $2h$. The current thickness of the quasisolid core is $2h_0$.

With allowance for the assumptions made we describe the flow by a system of differential equations of motion, continuity, and rheological state:

$$\frac{dp}{dx} = \frac{\partial \tau_{xy}}{\partial y} + \rho g \,, \quad \frac{\partial p}{\partial y} = 0 \,, \quad Q = 2 \int_0^h v_x \, dy \,, \tag{1}$$

The entire flow region in the interroller gap can be divided into two zones according to the character of the change in the pressure gradient and the velocity: in the first zone $x_0 < x < x_m$ the pressure gradient is positive $dp/dx > 0$, while in the zone of viscoplastic flow $h_0 < y < h$ the axial velocity of the quasisolid core is smaller than the peripheral velocity of the roller surface $\partial v_x/\partial y > 0$ (by convention we will call it the zone of counterflow); in the second zone $x_m < x < x_1$ the pressure gradient is negative $dp/dx < 0$, while in the zone of viscoplastic flow ($h_0 < y < h$) the velocity of the quasisolid core exceeds the velocity of the rollers $\partial v_x/dy < 0$

Volga Polytechnic Institute of Volgograd State Technical University, Volgograd, Russia. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 73, No. 4, pp. 787-791, July–August, 2000. Original article submitted June 8, 1999; revision submitted September 24, 1999.

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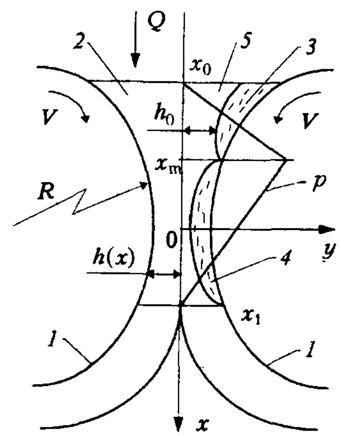

Fig. 1. Scheme of flow of a viscoplastic medium in a vertical gap between rollers: 1) rollers; 2) fluid; 3, 4) zones of viscoplastic shear flow; 5) quasisolid core.

(the zone of direct flow). At the interface between the zones $x = x_m$ and $h_0 = h$ the axial velocity of the quasisolid core is equal to the peripheral velocity of the rollers $v_x = V$, while the pressure curve has the bend $dp/dx|_{x=x_m-0} \neq dp/dx|_{x=x_m+0}$ if $\tau_0 \neq 0$. At the outlet cross section $x = x_1$ the quasisolid core touches the roller surfaces $h_0 = h$ and the axial velocity is uniform over the cross section $v_x = V$. At the inlet $x = x_0$ and at the outlet $x = x_1$ the pressure is equal to atmospheric pressure, and, without loss of generality, we set $p = 0$.

We supplement Eqs. (1) with the following boundary conditions:

the inlet

$$\dot{\gamma}=0\,, \quad |\tau_{xy}| < \tau_0\,, \quad \tau_{xy} = s\tau_0 + \eta\dot{\gamma}\,, \quad |\tau_{xy}| > \tau_0\,.$$

$$x = x_0\,, \quad p = 0\,, \tag{2}$$

the first zone (of counterflow) $x_0 < x < x_m$:

the adhesion condition

$$y = h\,, \quad v_x = V\,, \tag{3}$$

the boundary of the quasisolid core

$$y = h_0\,, \quad \dot{\gamma} = 0\,, \quad |\tau_{xy}| = \tau_0\,, \quad v_x = v_0\,, \tag{4}$$

the interface of the zones

$$x = x_m\,, \quad v_x = V\,, \quad h_0 = h\,, \tag{5}$$

the second zone (of direct flow) $x_m < x < x_1$:

the adhesion condition

$$y = h\,, \quad v_x = V\,, \tag{6}$$

the boundary of the quasisolid core

$$y = h_0\,, \quad \dot{\gamma} = 0\,, \quad |\tau_{xy}| = \tau_0\,, \quad v_x = v_0\,, \tag{7}$$

the outlet cross section

$$x = x_1\,,\quad p = 0\,,\quad v_x = V\,,\quad |\tau_{xy(y=h)}| = \tau_0\,, \tag{8}$$

the symmetry condition

$$x_0 < x < x_1\,,\quad y = 0\,,\quad \dot{\gamma} = 0\,,\quad \tau_{xy} = 0\,. \tag{9}$$

Integrating the equation of motion in (1) with account for condition (9), we have

$$\tau_{xy} = \left(\frac{dp}{dx} - \rho g\right) y\,. \tag{10}$$

The distribution of shear stresses over the gap width is linear. There is a symmetric region relative to the x axis where $|\tau_{xy}| \le \tau_0$ and the behavior of the fluid is similar to that of a solid. The current dimension of the quasisolid core h_0 is determined from Eq. (10) with account for Eqs. (4) and (7):

$$s\tau_0 = \left(\frac{dp}{dx} - \rho g\right) h_0\,. \tag{11}$$

Here and below the sign s indicates that the expression belongs to the first ($s = +1$) or the second zone ($s = -1$).

Considering Eqs. (10) and (11) and the equation of state from (1) simultaneously, we can write

$$\frac{\partial v_x}{\partial y} = \frac{s\tau_0}{\eta}\left(\frac{y}{h_0} - 1\right). \tag{12}$$

Integration of Eq. (12) with account for the adhesion conditions (3) and (6) gives

$$v_x = \frac{s\tau_0}{\eta}\left(\frac{y^2 - h^2}{2h_0} - y + h\right) + V\,. \tag{13}$$

The velocity of the quasisolid core v_0 is found from conditions (4) and (7):

$$v_0 = -\frac{s\tau_0\,(h_0 - h)^2}{2\eta h_0} + V\,. \tag{14}$$

The fluid flow rate in Eqs. (1) is composed of the axial flow rate of the quasisolid core and the flow rate in the zones of viscoplastic flow. With account for Eqs. (13) and (14) we have

$$Q = 2v_0 h_0 + 2\int_{h_0}^{h} v_x dy = 2Vh + \frac{s\tau_0}{\eta h_0}\,(h - h_0)\,(h_0^2 + hh_0 - 2h^2)\,. \tag{15}$$

Now we introduce the following dimensionless variables and parameters:

$$\{\xi, \xi_0, \xi_m, \lambda\} = \frac{\{x, x_0, x_m, x_1\}}{\sqrt{2RH_0}}\,;\quad \mathrm{St} = \frac{\rho g H_0^2}{\eta V}\,;\quad q = \frac{Q}{VH_0}\,;\quad \mathrm{S} = \frac{\tau_0 H_0}{\eta V}\,;$$

$$\mathrm{La} = \frac{pH_0^2}{\eta V\sqrt{2RH_0}}\,;\quad \zeta(\xi) = \frac{h_0(x)}{h(x)}\,. \tag{16}$$

The parabolic approximation [3] is taken for the roller surface:

$$h = H_0\,(1 + \xi^2)\,. \tag{17}$$

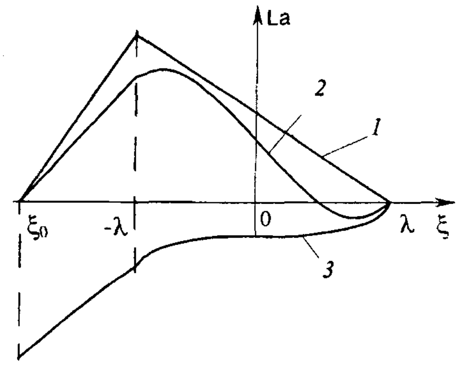

Fig. 2. Main characteristic cases of distribution of pressure over the length of the flow zone according to the parameters St and B: 1) $0 < \mathrm{St} < \mathrm{St}_*$; 2) $B < \mathrm{St} < \mathrm{St}_*$; 3) $\mathrm{St} > \mathrm{St}_*$.

Expression (15) with account for Eqs. (16) and (17) takes the form

$$q = 2(1+\xi^2) + sS\,\frac{3\zeta - \zeta^3 - 2}{3\zeta}\,(1-\xi^2)^2 . \tag{18}$$

The equation for the pressure (11) in the variables (16) will be represented as

$$\frac{d\mathrm{La}}{d\xi} = \mathrm{St} + \frac{sS}{\zeta\,(1+\xi^2)} . \tag{19}$$

At the outlet cross section, $\xi = \lambda$, $\zeta = 1$, and, in conformity with Eq. (18), $q = 2(1+\lambda^2)$. Equation (19) yields the boundary value of the pressure gradient $d\mathrm{La}/d\xi = \mathrm{St} - S/(1+\lambda^2)$. On the other hand, at the cross section $\xi = \xi_m$ the quasisolid core also touches the roller surfaces ($\zeta = 1$) and, according to Eq. (14), the velocity of the core is $v_0 = V$. Here, for the flow rate we have $q = 2(1+\xi_m^2)$ from Eq. (18). Comparing the expressions for the flow rate at the cross sections $\xi = \lambda$ and $\xi = \xi_m$, we obtain $\xi_m = -\lambda$.

The solution of Eqs. (18) and (19) with account for the boundary condition (8) and the equality $s = \mathrm{sign}\,(\xi - \lambda)$ can be represented in integral form:

$$\mathrm{La} = \mathrm{St}\,(\xi - \lambda) + \mathrm{S}\int_{\lambda}^{\xi} \frac{\mathrm{sign}\,(\xi - \lambda)}{\zeta\,(1+\xi^2)}\,d\xi ,$$

$$\zeta = 2r\cos\left(\frac{\pi + \varphi}{3}\right), \quad \varphi = \arccos\left(\frac{1}{r^3}\right), \quad r = \sqrt{\left(1 + \frac{2\,|\lambda^2 - \xi^2|}{\mathrm{S}\,(1+\xi^2)^2}\right)} . \tag{20}$$

Here the function $\zeta(\xi)$, as the solution of the cubic equation (18), is determined for a negative value of the discriminant (it is always negative) [4].

The parameter ξ_0 is found from condition (2):

$$\mathrm{St}\,(\xi_0 - \lambda) + \mathrm{S}\int_{\lambda}^{\xi_0} \frac{\mathrm{sign}\,(\xi - \lambda)}{\zeta\,(1+\xi^2)}\,d\xi = 0 .$$

An analysis of Eq. (19) shows that the character of the pressure distribution over the channel length is determined by the relation of the dimensionless combinations St and $B = S(1+\lambda^2)$. Here we can single out three cases.

1. A slight influence of the intrinsic-weight forces. The pressure distribution has the form of curve 1 in Fig. 2. The condition $0 \le \mathrm{St} < B$ is fulfilled. The pressure gradient satisfies the conditions $d\mathrm{La}/d\xi \ge \mathrm{St} + B$ on the section $\xi_0 < \xi < \lambda$ and $d\mathrm{La}/d\xi \le \mathrm{St} - B$ on the section $|\xi| < \lambda$. At the cross section $\xi = -\lambda$ the function $d\mathrm{La}/d\xi$ suffers a discontinuity, and the pressure curve has a bend if $S \ne 0$. At the outlet cross section $\xi = \lambda$ the pressure gradient is not equal to zero and the cavitational condition $d\mathrm{La}/d\xi = 0$ that holds for the viscous fluid is replaced by the condition of cessation of the flow $d\mathrm{La}/d\xi = \mathrm{St} - B$. The regime boundary is characterized by the equality $\mathrm{St} = B$, which is obtained if in Eq. (19) we take $\zeta = 1$, $\xi = \lambda$, $s = -1$, and $d\mathrm{La}/d\xi = 0$. At the points $\xi = \pm\lambda$, $d\mathrm{La}/d\xi = 0$, and on the interval $|\xi| < \lambda$, $d\mathrm{La}/d\xi < 0$.

2. The regime of a moderate influence of the forces of gravity. The pressure distribution has the form of curve 2 in Fig. 2. The Stokes number lies in the interval $B < \mathrm{St} < \mathrm{St}_*$. We can find the boundary value St_* using the conditions $\xi = 0$ and $d\mathrm{La}/d\xi = 0$ for expressions (19) and (20):

$$\mathrm{St}_* = \frac{\mathrm{S}}{\zeta_*}, \quad \zeta_* = 2r_* \cos\left(\frac{\pi + \varphi_*}{3}\right), \quad \varphi_* = \arccos\left(\frac{1}{r_*^3}\right), \quad r_* = \sqrt{\left(1 + \frac{2\lambda^2}{\mathrm{S}}\right)}. \tag{21}$$

It is seen from Fig. 2 that on the section $|\xi| < \lambda$ the function La has two extrema located symmetrically relative to the cross section $\xi = 0$. Here the minimum in the vicinity of $\xi = \lambda$ assumes rarefaction $\mathrm{La} < 0$. In the Newtonian case ($S = 0$) or without allowance for the forces of gravity ($\mathrm{St} = 0$) the noted effect disappears.

3. The regime of a strong influence of the forces of gravity. The pressure gradient on the entire section is positive (curve 3 in Fig. 2). The relation $\mathrm{St} > \mathrm{St}_*$ is fulfilled. The boundary condition (2) ($\xi = \xi_0$, $\mathrm{La} = 0$) for the pressure at the inlet is not fulfilled. Over the entire zone of the flow the pressure is vacuum gauge. In practice, this regime can be implemented by providing either a lowered pressure above the fluid surface or an excess pressure at the outlet, i.e., at the cross section $\xi = \lambda$. The pumping effect of the rollers, caused by the forces of viscous friction, is manifested. In particular, this effect forms the basis for the operation of a roller extruder [2] and for the process of deposition of a coating on a metal plate [5].

It should be noted that in all of the above-indicated regimes the character of the velocity field is retained: at the cross sections $|\xi| = \lambda$ the quasisolid core touches the roller surfaces, while in the zones of viscoplastic flow the inequalities $\xi_0 < \xi < -\lambda$ and $\partial v_x/\partial y > 0$ are fulfilled in the region of counterflow and $|\xi| < \lambda$ and $\partial v_x/\partial y < 0$ are fulfilled in the region of direct flow.

Let us find the characteristic ranges of shear rates and shear stresses necessary for determining the rheological constants. The lowest tangential stress and shear rate on the flow axis $y = 0$ are int $(\tau_{xy}) = 0$ and int $(\dot\gamma) = 0$. Correspondingly, the greatest tangential stress sup (τ_{xy}) and shear rate sup $(\dot\gamma)$ occur at the points $x = 0$, $y = \pm H_0$ (or $\xi = 0$, $\zeta = \pm h_0/H_0$). Using the equation of state from (1), with account for Eqs. (11), (16), (17), (19), and (20) we can write

$$\sup(\dot\gamma) = \frac{\tau_0}{\eta}\left(\frac{1}{\zeta_*} - 1\right), \quad \sup(\tau_{xy}) = \frac{\tau_0}{\zeta_*},$$

where ζ_* is determined according to (21).

Now we find the integral parameters of the flow. The frictional force acting on the roller surface of unit length from the side of the fluid with account for Eqs. (10), (11), and (16) is determined by the integral

$$F = \int_{x_0}^{x_1} \tau_{xy}\Big|_{y=h} dx = \tau_0 \sqrt{2RH_0} \int_{\xi_0}^{\lambda} \frac{d\xi}{\zeta},$$

where the function $\zeta(\xi)$ is described by relations (20).

The magnitude of the thrust force, calculated per unit working length of the roller, with account for Eqs. (16) and (19) is found by integration by parts:

$$T = \int_{x_0}^{x_1} p dx = -\frac{2VR\eta}{H_0} \int_{\xi_0}^{\lambda} \left[\mathrm{St} + \frac{\mathrm{S}\,\mathrm{sign}\,(\xi - \lambda)}{(1 + \xi^2)\,\zeta} \right] \xi d\xi \,.$$

The technological power of the process referred to unit working length of the roller is equal to $N = 2VF$.

NOTATION

p, pressure; τ_{xy}, tangential stress; ρ, fluid density; g, free-fall acceleration; Q, volumetric flow rate of the fluid per unit working length of the roller; v_x, axial velocity component; y, x, Cartesian coordinates; $\dot{\gamma} = \partial v_x/\partial y$, rates of shear; τ_0, limiting shear stress; $s = \mathrm{sign}\,(\dot{\gamma}) = \pm 1$, sign of the rate of shear; η, plastic viscosity; q, dimensionless flow rate; S, Il'yushin number; St, Stokes number; St_*, critical value of the Stokes number; ξ, Haskell dimensionless variable; 2ζ, dimensionless current thickness of the quasisolid core; La, Lagrange number; x_0, x_1, and x_m, coordinates of the inlet, the outlet, and the point of maximum pressure; ξ_0, ξ_m, and λ, dimensionless coordinates of the inlet, the point of the maximum, and the outlet from the gap; φ, r, and β, functions; F, frictional force; T, thrust force; N, consumed power.

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